Dequantization Barriers for Guided Stoquastic Hamiltonians

Shrinidhi T S

Based on joint work with *Yassine Hamoudi* and *Yvan Le Borgne*

https://arxiv.org/abs/2602.23183

Ground state information of a many-body system

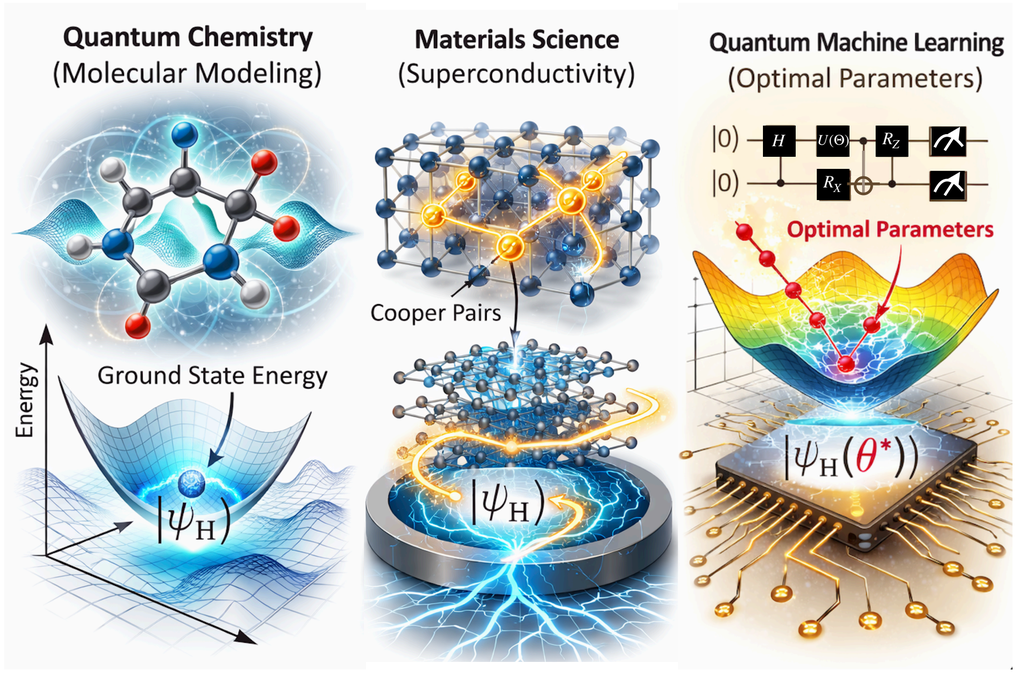
m-qubit Hamiltonian
$H \in \mathbb{C}^{2^m \times 2^m}$

Ground state $|\psi_H\rangle$,
Ground energy λ_H, etc.,.

Generally hard without extra information
even for quantum computers

Ex: Computing $\lambda_H \pm \delta$ is QMA-Hard for *local* H

Hardness quantified by:
Number of gates, number of qubits, queries etc.,.



Images generated by ChatGPT

Preparing the ground state $|\psi_H\rangle$

Given access to H, prepare the ground state $|\psi_H\rangle$

More g*eneral* *than* estimating $\lambda_H \Longrightarrow$ believed to be **hard** even for quantum

But, significantly easier when given the following extra information

Guiding state $|\psi_{\text{in}}\rangle$: $\langle\psi_{\text{in}}|\psi_H\rangle$ = large

Notion similar to

Ansätz - QAOA, Variational Quantum Algorithms

Warm starts - Markov chain Monte Carlo

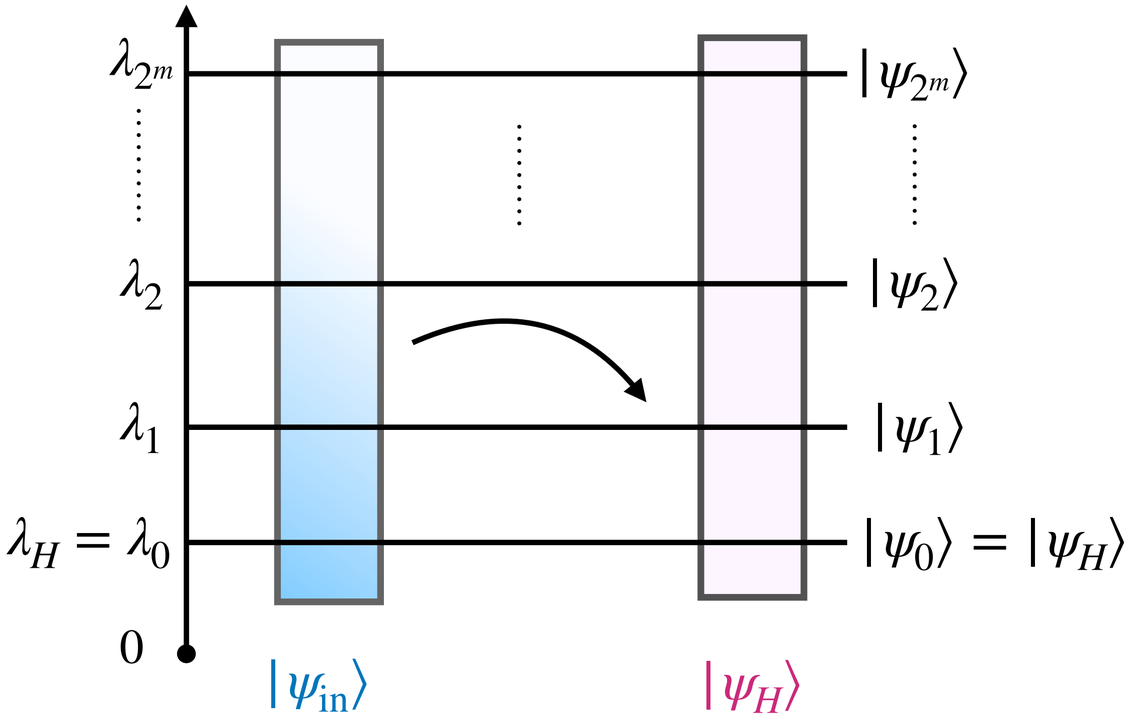
Intermediate states - Adiabatic Quantum Computation

Complexity of guided ground state preparation

Guided Ground State Preparation (GGSP)

Given access to H and copies of a *guiding* state $|\psi_{\text{in}}\rangle$, prepare $|\psi_H\rangle$

Efficient quantumly when $\langle\psi_{\text{in}}|\psi_H\rangle \geq 1/\text{poly}(m)$



1) Apply QPE on $|\psi_{\text{in}}\rangle$

2) Measure the energy register $\rightarrow |\psi_H\rangle$

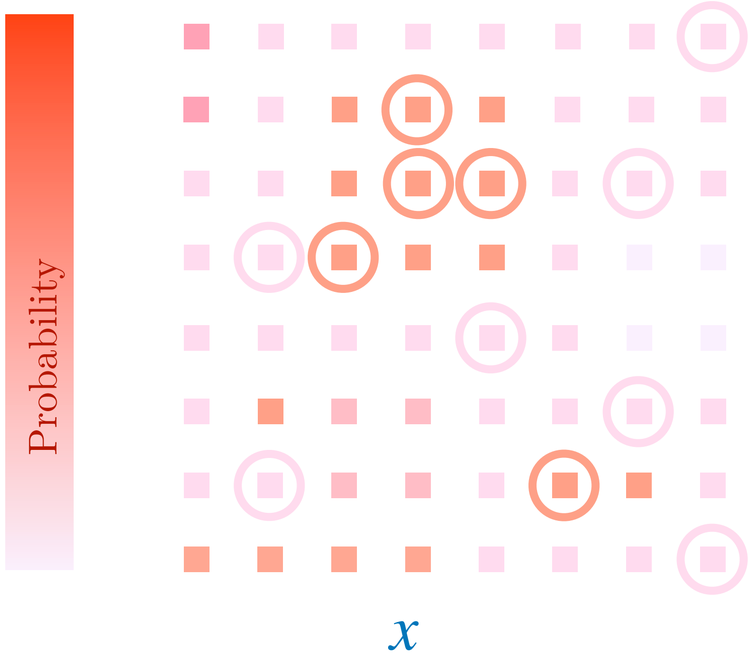
Standard assumptions on $H \rightarrow$
runs in poly (m) time, poly(m) copies of $|\psi_{\text{in}}\rangle$

Classical Complexity of Guided Ground State Problems

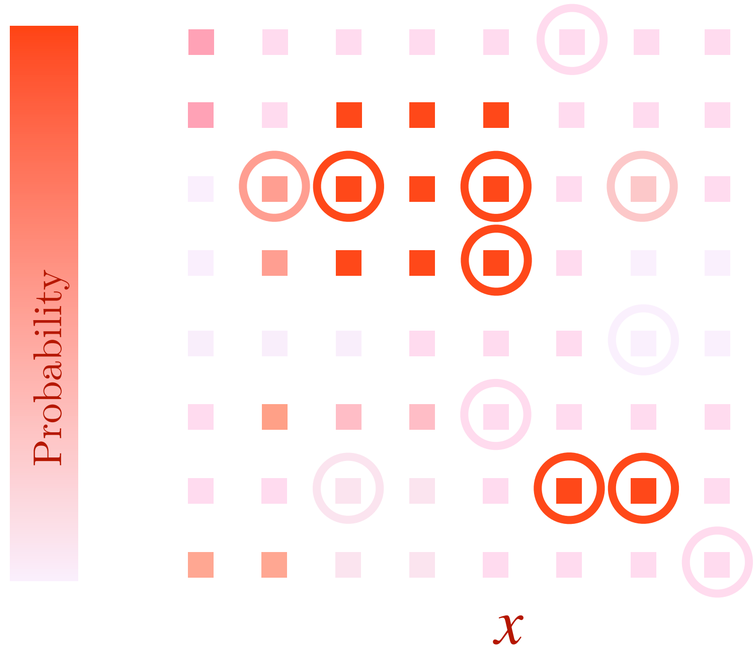
What is copies of $|\psi_{\text{in}}\rangle$ classically?

What is preparing $|\psi_H\rangle$ classically?

$|\langle x|\psi_{\text{in}}\rangle|^2$ **Sampling access** to $|\psi_{\text{in}}\rangle$



$|\langle x|\psi_H\rangle|^2$ **Sample from** $|\psi_H\rangle$



Another common access to $|\psi_{\text{in}}\rangle$: **query access** to amplitudes of $|\psi_{\text{in}}\rangle$

Classical Complexity of Ground State Problems

1) When **sampling + query** access to $|\psi_{\text{in}}\rangle$

Classical Algorithms for Constant Approximation of the Ground State Energy of Local Hamiltonians

Author François Le Gall

Part of: Volume: 33rd Annual European Symposium on Algorithms (ESA 2025)

Computing $\lambda_H \pm \delta$ classically for *guided* **local** H takes poly $\left(1/\langle\psi_{\text{in}}|\psi_H\rangle^{1/\delta}, m\right)$ time

2) When $\langle x|\psi_H\rangle / \langle y|\psi_H\rangle$ is efficiently computable

Quantum
the open journal for quantum science

PAPERS PERSPECTI

A rapidly mixing Markov chain from any gapped quantum many-body system

Sergey Bravyi[1], Giuseppe Carleo[2], David Gosset[3,4], and Yinchen Liu[3,4] 2023

Preparing $|\psi_H\rangle$ is *classically* easy for any *gapped, local* H

This talk: **only** sampling access to $|\psi_{\text{in}}\rangle$ for classical algorithms

First step: Stoquastic GGSP

H is **stoquastic** (or *sign-free*) if its off-diagonal terms are non-positive

Perron-Frobenius theorem: $\langle x | \psi_H \rangle \geq 0$ for all x

Example: Classical optimisation functions - diagonal H,
Transverse Field Ising Model, etc.,.

Classical simulations like Quantum Monte Carlo → usually efficient for stoquastic H

Is GGSP *classically* easy when H is *stoquastic*?

Is GGSP *classically* easy when H is *stoquastic*?

1) $|\psi_{\text{in}}\rangle$ and $|\psi_H\rangle$ are point-wise close and query access to $|\psi_{\text{in}}\rangle$

Complexity of Stoquastic Frustration-Free Hamiltonians

Authors: Sergey Bravyi and Barbara Terhal | AUTHORS INFO & AFFILIATIONS

https://doi.org/10.1137/08072689X 2010

GGSP is *classically* easy for

gapped + stoquastic + frustration free

2) $\langle\psi_{\text{in}}|\psi_H\rangle$ is large and sample-only access to $|\psi_{\text{in}}\rangle$

(Sub)Exponential advantage of adiabatic Quantum computation with no sign problem

Authors: András Gilyén, Matthew B. Hastings, Umesh Vazirani | Authors Info & Claims

STOC 2021: Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing • Pages 1357 - 1369

∃ a stoquastic H: *classically* **hard** to prepare $|\psi_H\rangle$ given sampling access to a **particular** $|\psi_{\text{in}}\rangle$ whereas it is *quantumly* easy to prepare $|\psi_H\rangle$

Is GGSP *classically* easy when H is *stoquastic*?

Our results:

This talk

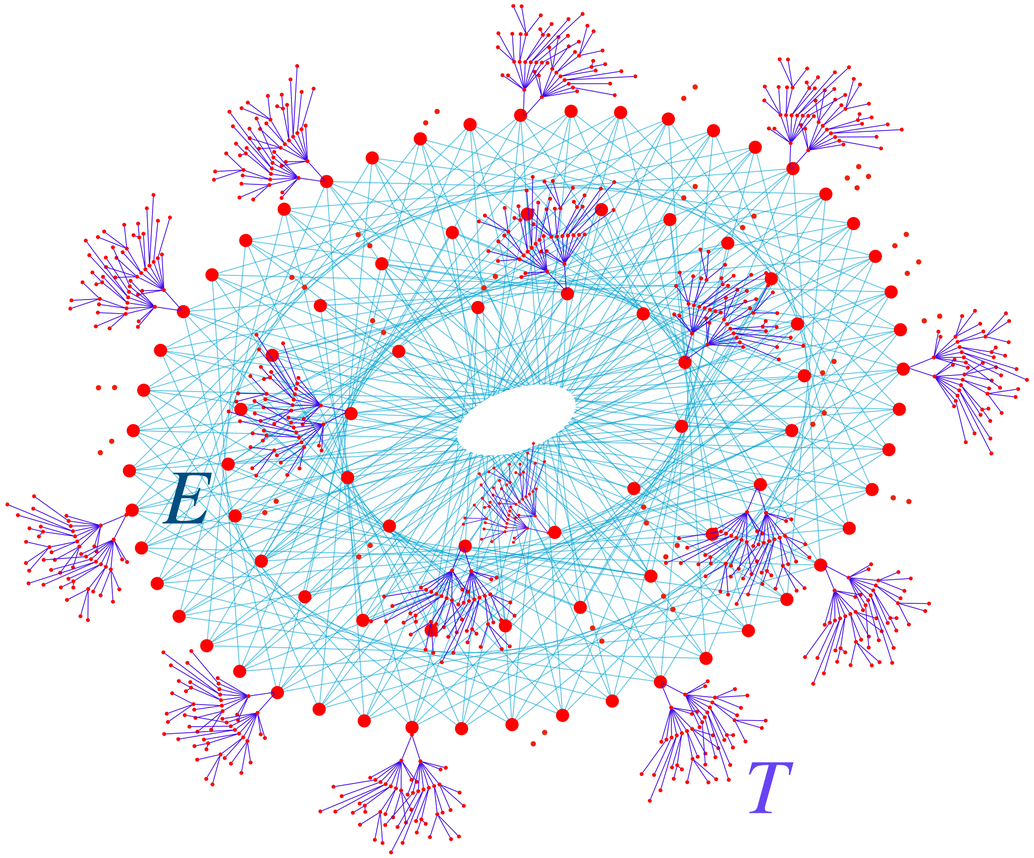
1. ∃ a stoquastic H: *classically* **hard** to prepare $|\psi_H\rangle$ given copies of an **arbitrary** $|\psi_{\text{in}}\rangle$, whereas it is *quantumly* easy

To be updated on arXiv soon!

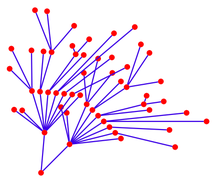
2. ∃ a stoquastic H: *classically* **hard** to estimate λ_H given copies of a **particular** $|\psi_{\text{in}}\rangle$ very close to $|\psi_H\rangle$, whereas it is *quantumly* easy

*Both hardness results are in terms of number of queries to a resource

Proof sketch: Construction of H



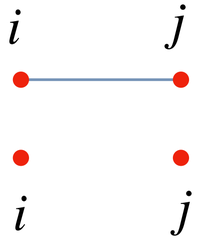
G on 2^m vertices



Tree T - On all vertices

$$H = -A_G$$

$$\langle i|H|j\rangle = \begin{cases} -1 & \text{if} \\ 0 & \text{if} \end{cases}$$



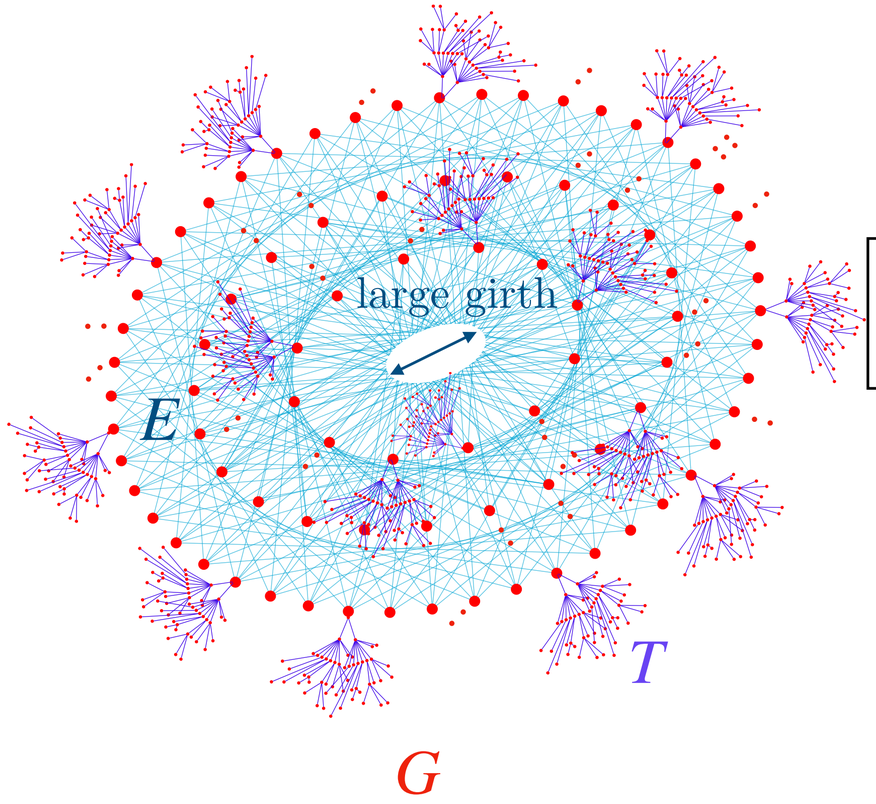
Stoquastic

$$|\psi_H\rangle = \text{top eigenvector of } A_G$$

Every vertex has poly(m) degree

Non-local, sparse and stoquastic H

Proof sketch: Construction of G



Very huge graph!

2^m vertices in G and $|V_{\text{Trees}}| \gg |V_E|$

E: expander with girth $\geq \text{poly}(m)$ and spectral gap $\geq \frac{1}{\text{poly}(m)}$

$+$ Tree T

G: Locally tree like!

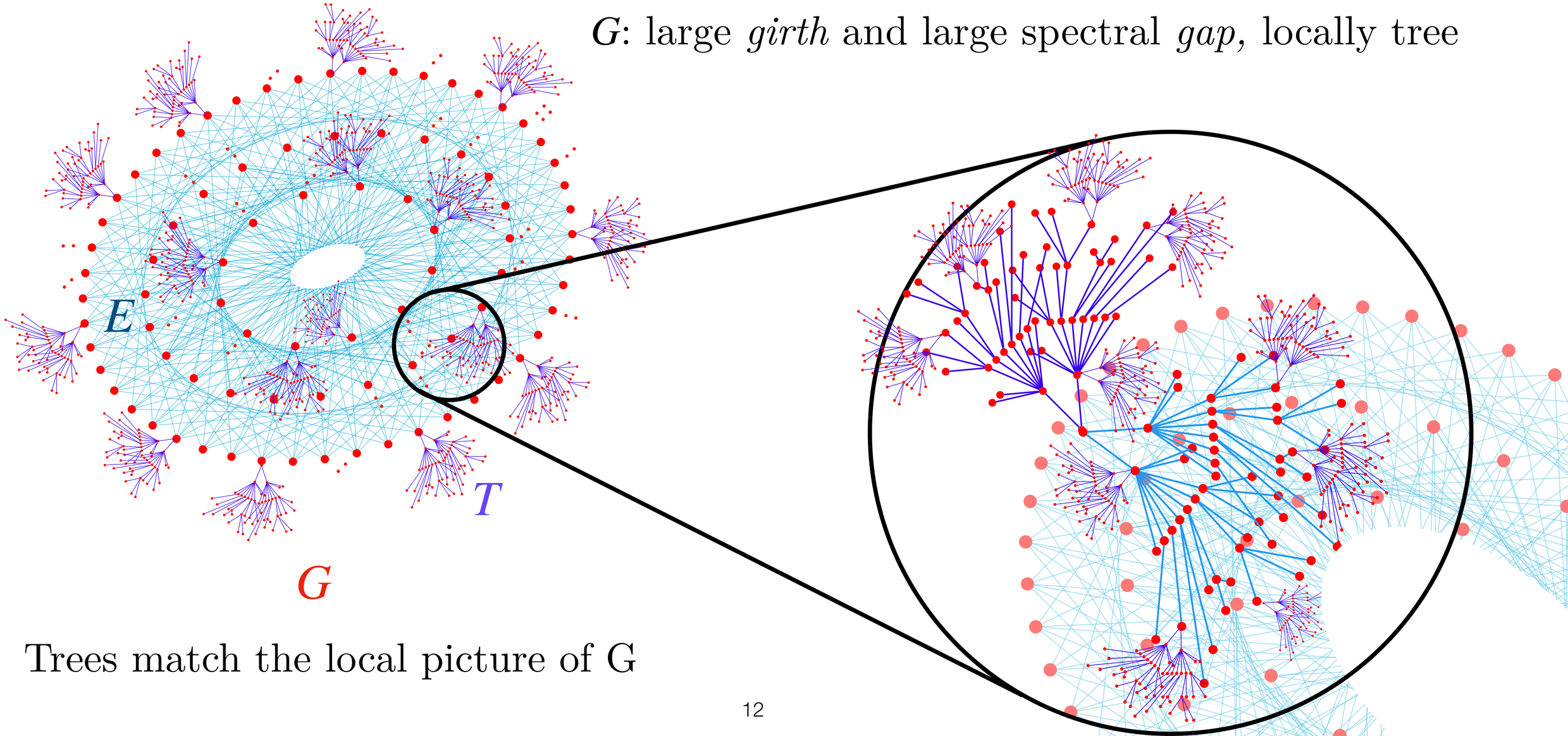
G: large girth and large spectral gap

Girth: length of the smallest cycle

Spectral gap of G: difference between the two largest eigenvalues of A_G

Proof sketch: Construction of H

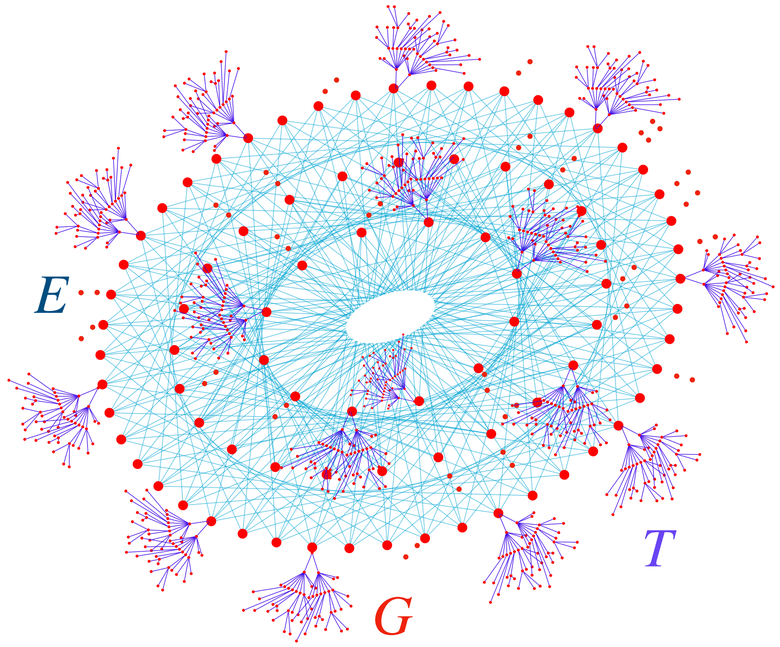
G: large *girth* and large spectral *gap*, locally tree



Trees match the local picture of G

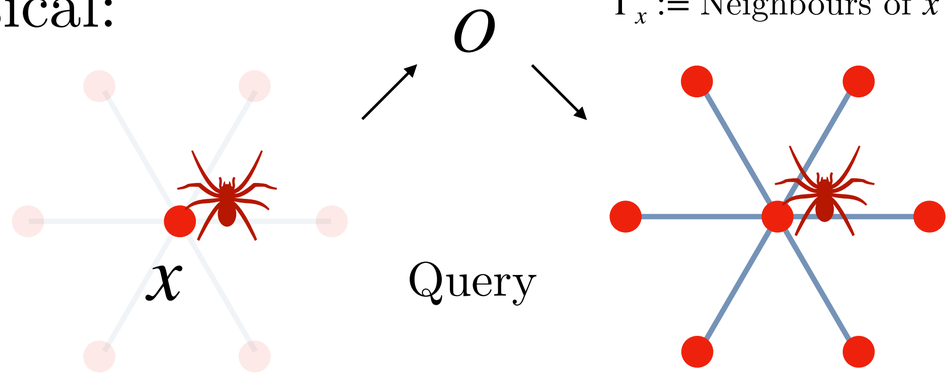
Proof sketch: Construction of H

$H = -A_G$: sparse, large *spectral gap*



Oracular access to H

Classical:



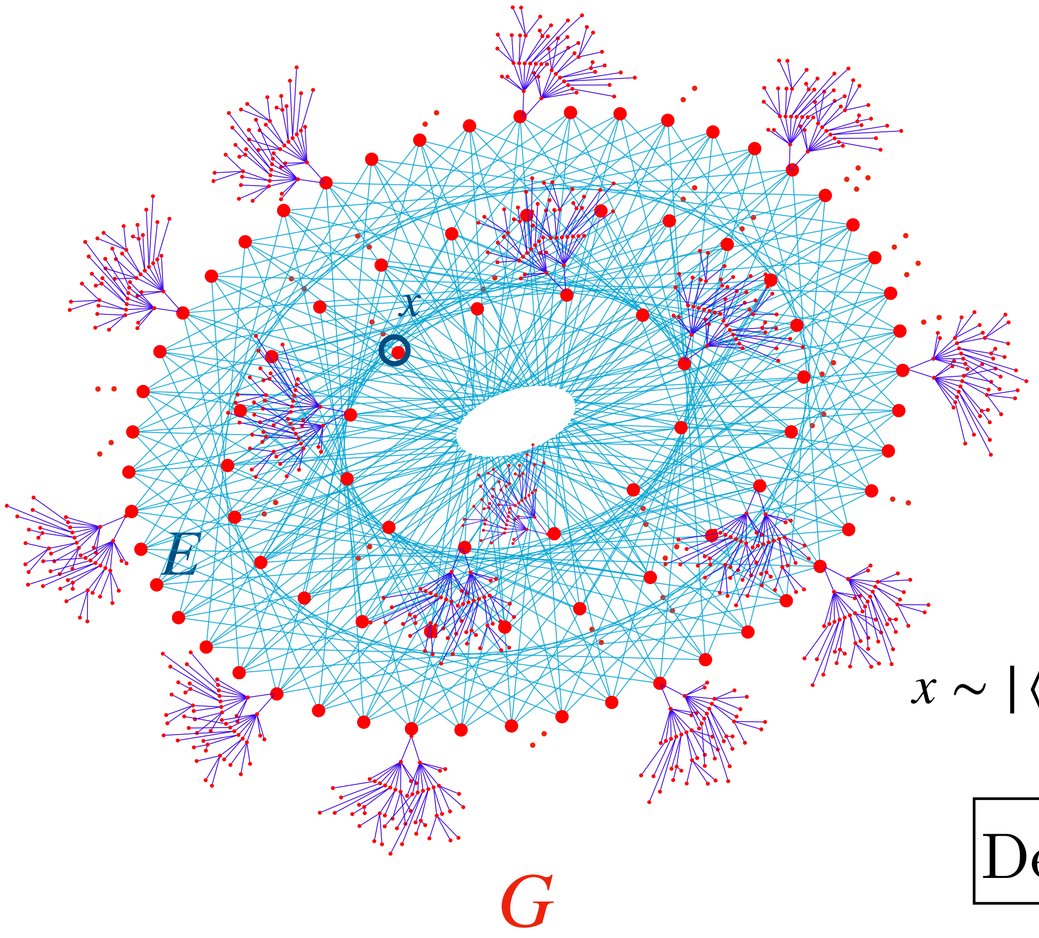
Quantum:

$$O|x\rangle|0\rangle \mapsto |x\rangle|\Gamma_x\rangle$$

Hard/Easy - More/Less queries to O

✓ ∃ a *quantum* algorithm for GGSP performing poly(m) queries

Concentration property of $|\psi_H\rangle$



Concentration Property of $|\psi_H\rangle$

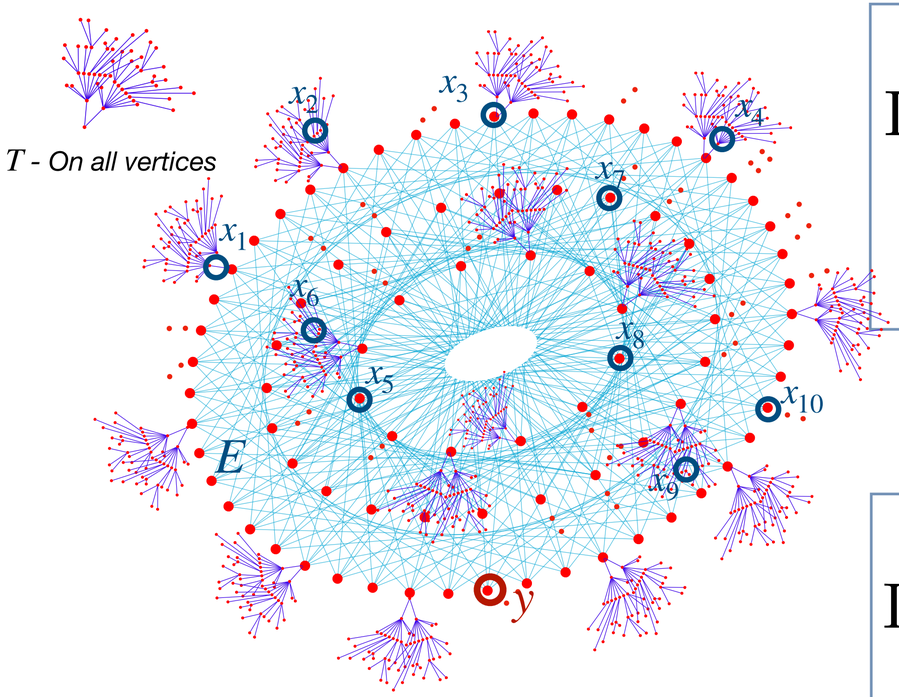
$|\langle \text{Trees} | \psi_H \rangle|^2 = o(1)$ Very tiny

$|\langle \text{Expander} | \psi_H \rangle|^2 = 1 - o(1)$ Very large

$x \sim |\langle x | \psi_H \rangle|^2 \implies x \in$ Expander, with high probability

Despite far more vertices lying in the trees!

Classical Hardness Proof sketch: Localization



Localization Property of $|\psi_H\rangle$

Let $x_1, x_2 \ldots x_t \sim \left(|\langle x_i | \psi_H \rangle|^2 \right)^{\otimes t}$. If $y \sim |\langle y | \psi_H \rangle|^2$, then y is **far** from all x_is w.h.p.

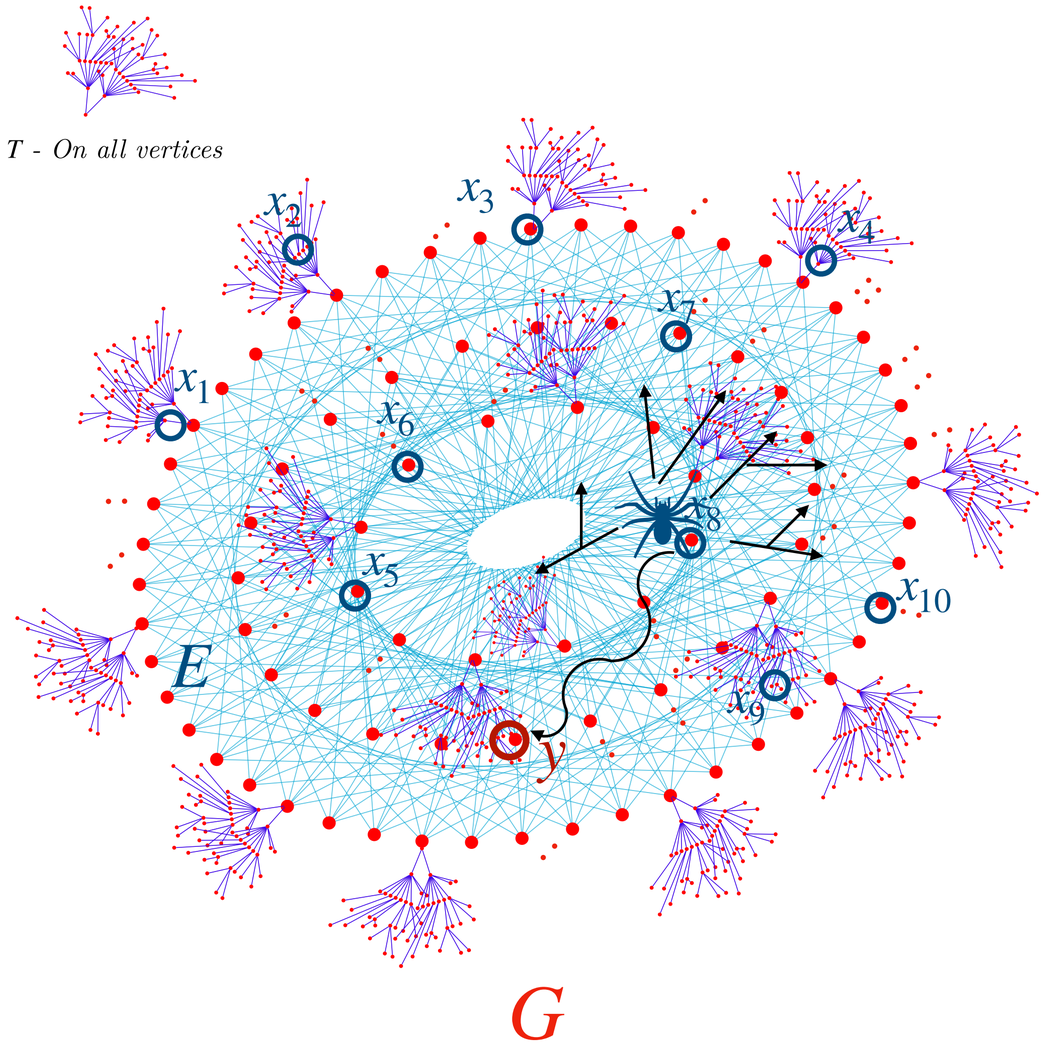
Localization Property for **any** $|\psi_{\text{in}}\rangle$

Let $x_1, x_2 \ldots x_t \sim \left(|\langle x_i | \psi_{\text{in}} \rangle|^2 \right)^{\otimes t}$. If $y \sim |\langle y | \psi_H \rangle|^2$, then y is **far** from all x_is w.h.p.

Holds for arbitrary guiding states!

Any classical algorithm solving GGSP must ***output*** a y ***far*** from all x_is

Proof sketch: Localization



Any classical algorithm solving GGSP must ***output*** a y ***far*** from all x_is

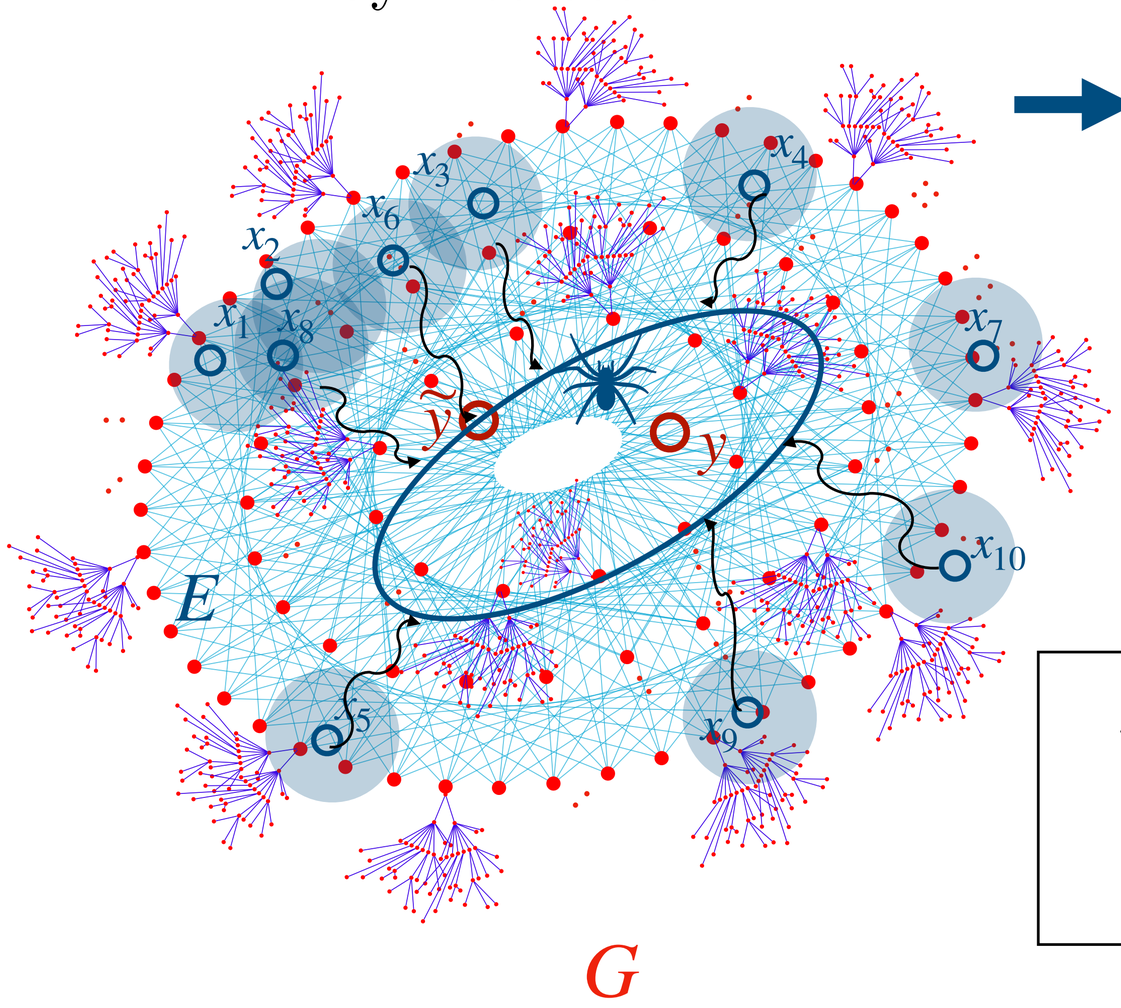
Only oracle access to H

Classical algorithms can perform only ***local exploration of*** G from x_i s

Any classical algorithm solving GGSP must ***explore*** a y ***far*** from all x_is

Proof sketch: Localization

Explorations from nearby x_is may intersect



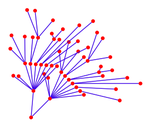
But y is ***far*** from all x_is + G has large girth!

Explorations farther from the samples remain disjoint for a **long** distance!

Any classical algorithm solving GGSP must ***explore*** a y which is ***far*** from all x_is

Any classical algorithm solving GGSP must ***explore*** a $\tilde{y}$ which is ***far*** from some vertex, *through a disjoint region*

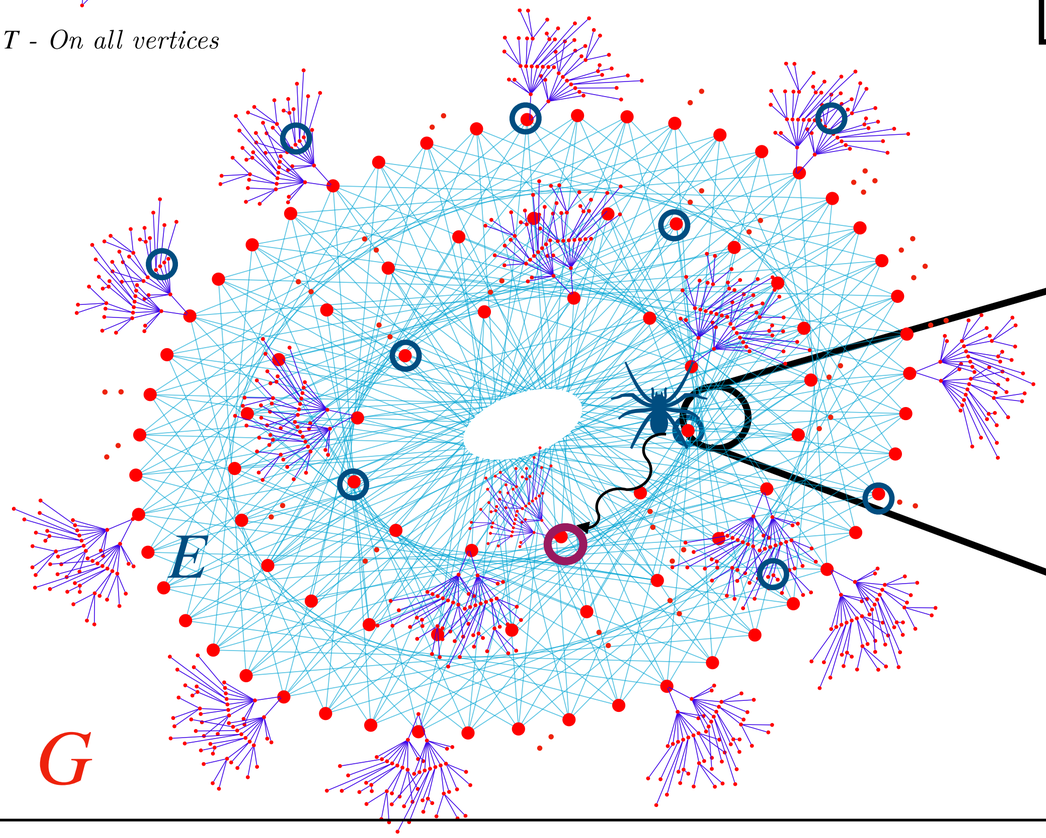
Proof sketch: How difficult is it to find a far y?



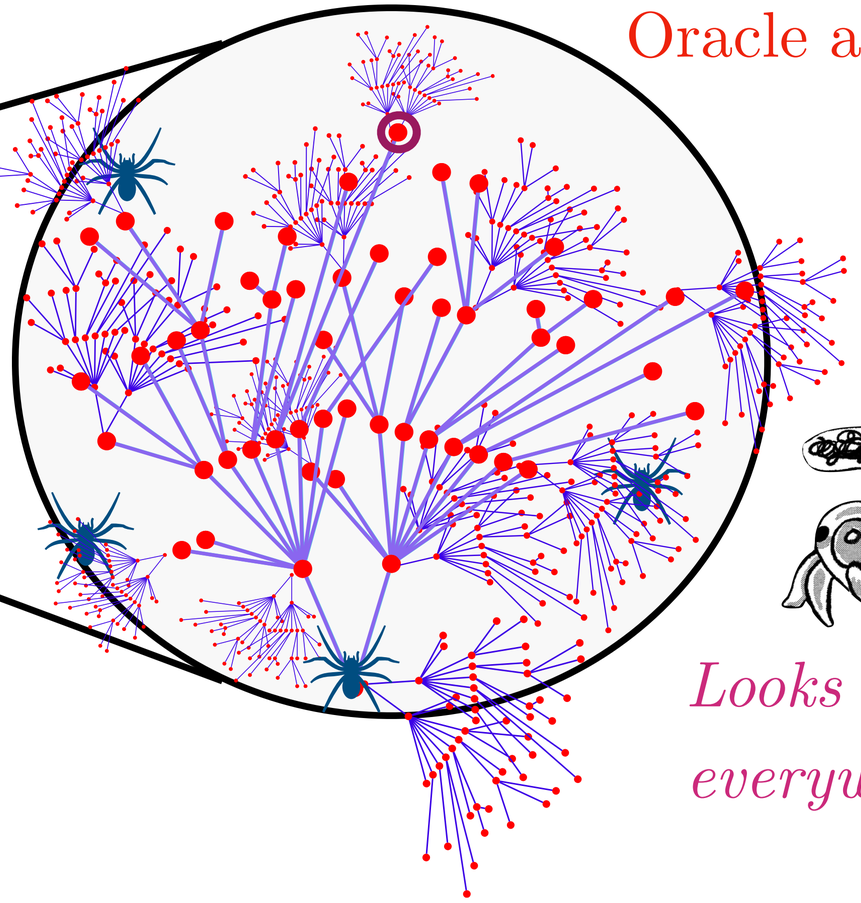
T - On all vertices

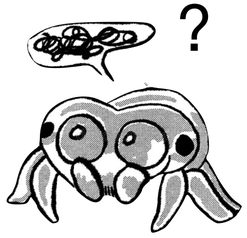
This is very difficult! ✘

Any classical algorithm solving GGSP must ***explore*** a $\tilde{y}$ which is ***far*** from some vertex, *through a disjoint region*



Oracle access!





Looks same everywhere!!

Vanishing probability even with sub-exponentially many samples and queries

Conclusion

Our result

$\exists$ a stoquastic H: *classically* **hard** $\left(2^{m^c}, c < 1\right)$ to prepare $|\psi_H\rangle$, given copies of an **arbitrary** $|\psi_{\text{in}}\rangle$ whereas it is *quantumly* easy $\left(\text{poly}(m)\right)$

Evidence against *dequantization* of GGSP given sampling access to arbitrary $|\psi_{\text{in}}\rangle$

Evidence against *dequantization* of *adiabatic paths* through our H, even *non-stoquastic* ones!

Open questions

1. Which graph decorations hinder exploration while preserving the gap?
Explicit high-girth expanders via Ramanujan graphs?

2.What is the power of Stoquastic Adiabatic computation?

3. Extend hardness to local H?
Sample/advice + query access to $|\psi_{\text{in}}\rangle$? QCMA vs QMA?