Dequantization Barriers for Guided Stoquastic Hamiltonians

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Based on joint work with *Yassine Hamoudi* and *Yvan Le Borgne*

https://arxiv.org/abs/2602.23183

LaBRI

Ground state information of a many-body system

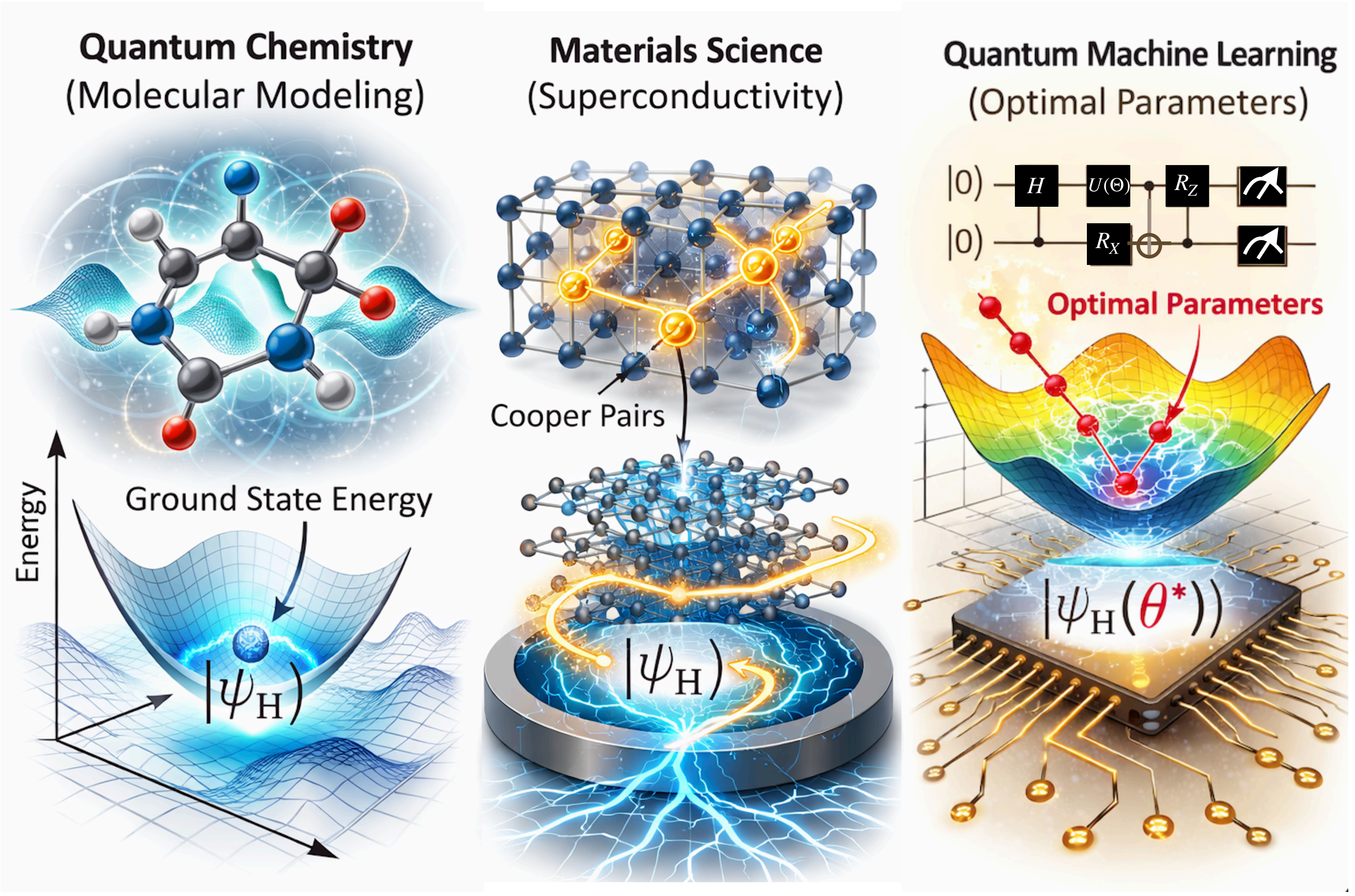
m-qubit Hamiltonian
$H \in \mathbb{C}^{2^m \times 2^m}$ → Ground state $|\psi_H\rangle$,
Ground energy λ_H, etc.,.

Generally hard without extra information
even for quantum computers

Ex: Computing $\lambda_H \pm \delta$ is QMA-Hard for *local* H

Hardness quantified by:

Number of gates, number of qubits, queries etc.,.



Images generated by ChatGPT

Preparing the ground state $|\psi_H\rangle$

Given access to H, prepare the ground state $|\psi_H\rangle$

More *general* *than* estimating $\lambda_H \implies$ believed to be **hard** even for quantum

But, significantly easier when given the following extra information

Guiding state $|\psi_{\text{in}}\rangle$: $\langle\psi_{\text{in}}|\psi_H\rangle$ = large

Notion similar to

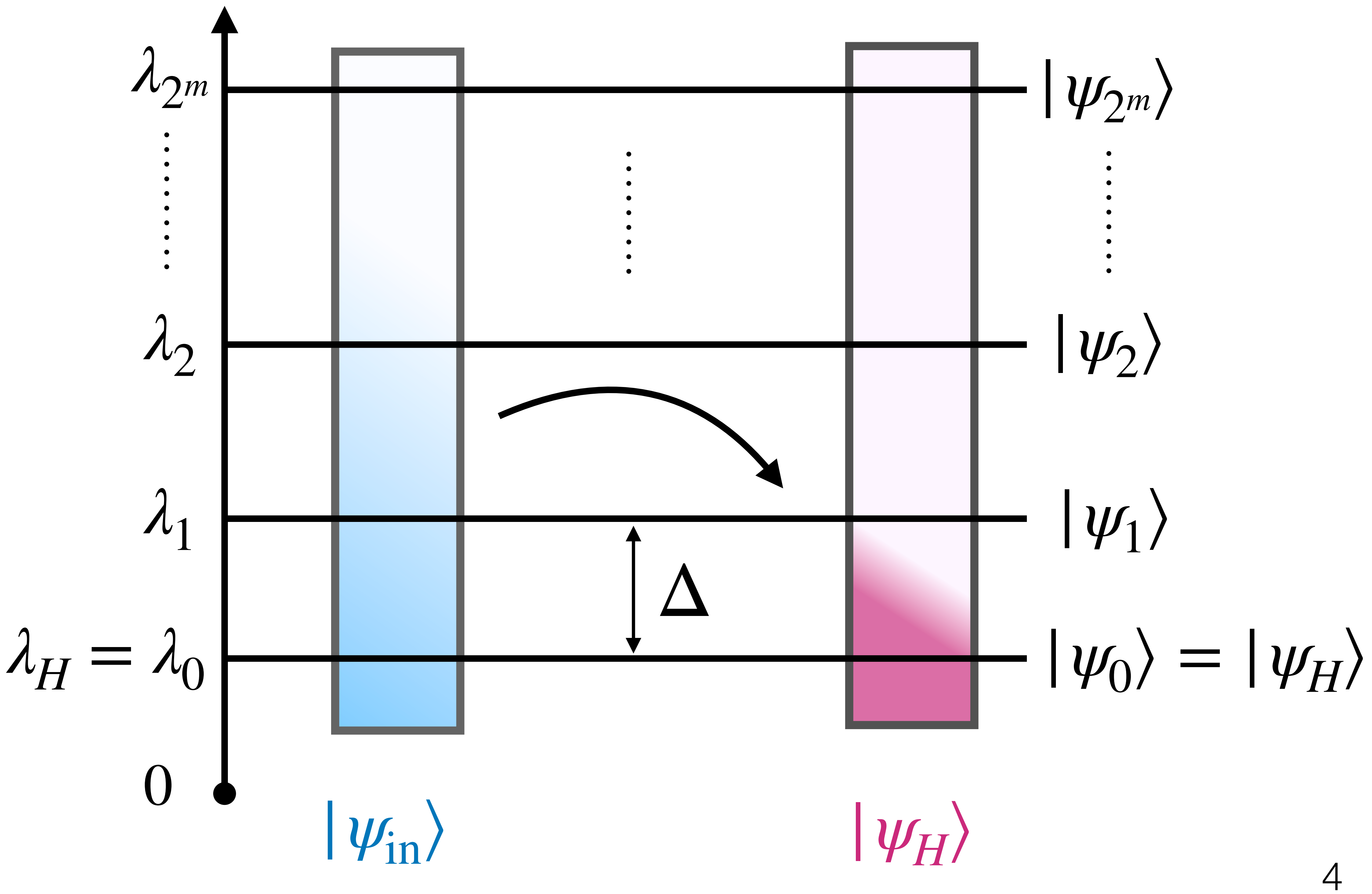
- *Ansätz* - QAOA, Variational Quantum Algorithms
- *Warm starts* - Markov chain Monte Carlo
- *Intermediate states* - Adiabatic Quantum Computation

Complexity of guided ground state preparation

Guided Ground State Preparation (GGSP)

Given access to H and a *guiding* state $|\psi_{\text{in}}\rangle$, prepare the ground state $|\psi_H\rangle$

GGSP is quantumly *efficient* for sufficiently large overlap



Filter out $|\psi_H\rangle^\perp$ from $|\psi_{\text{in}}\rangle$ using QPE

Quantum Algorithm using $|\psi_{\text{in}}\rangle$, prepares $|\psi_H\rangle$ in $\text{poly}\left(m, \frac{1}{\Delta}, \frac{1}{\langle\psi_{\text{in}}|\psi_H\rangle}\right)$ time*

**Under some reasonable assumptions*

Complexity of Guided Ground State Preparation

Given access to H and a *guiding* state $|\psi_{\text{in}}\rangle$, prepare the ground state $|\psi_H\rangle$

GGSP is efficiently solvable on *quantum computer* for sufficiently large overlap

Is there an efficient classical algorithm for GGSP?

François Le Gall, 2025

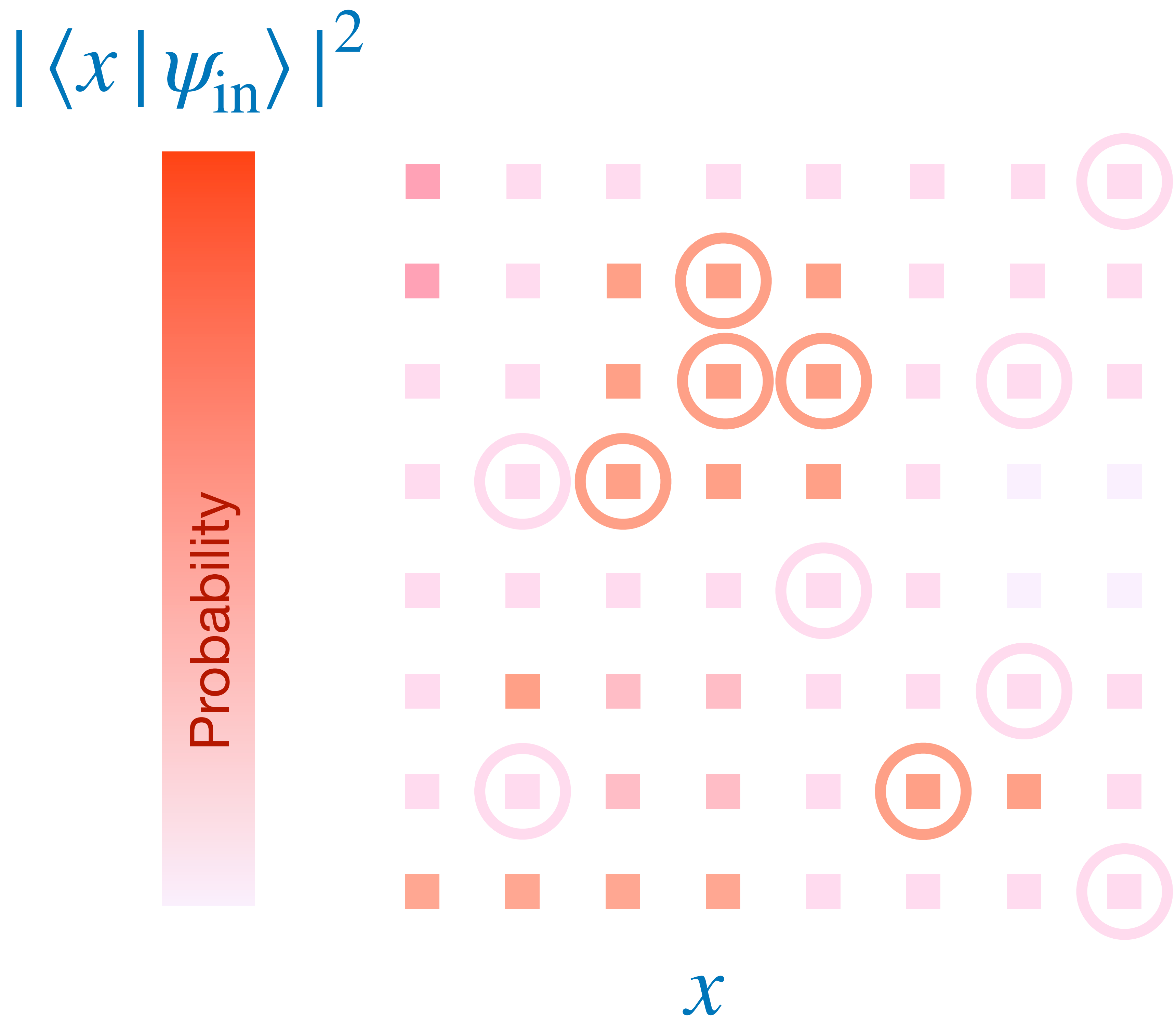
Computing $\lambda_H \pm \delta$ for *guided* **local** H classically takes $\text{poly}\left(\frac{1}{\langle\psi_{\text{in}}|\psi_H\rangle^{1/\delta}}, m\right)$ time

Can GGSP be **dequantized** as well?

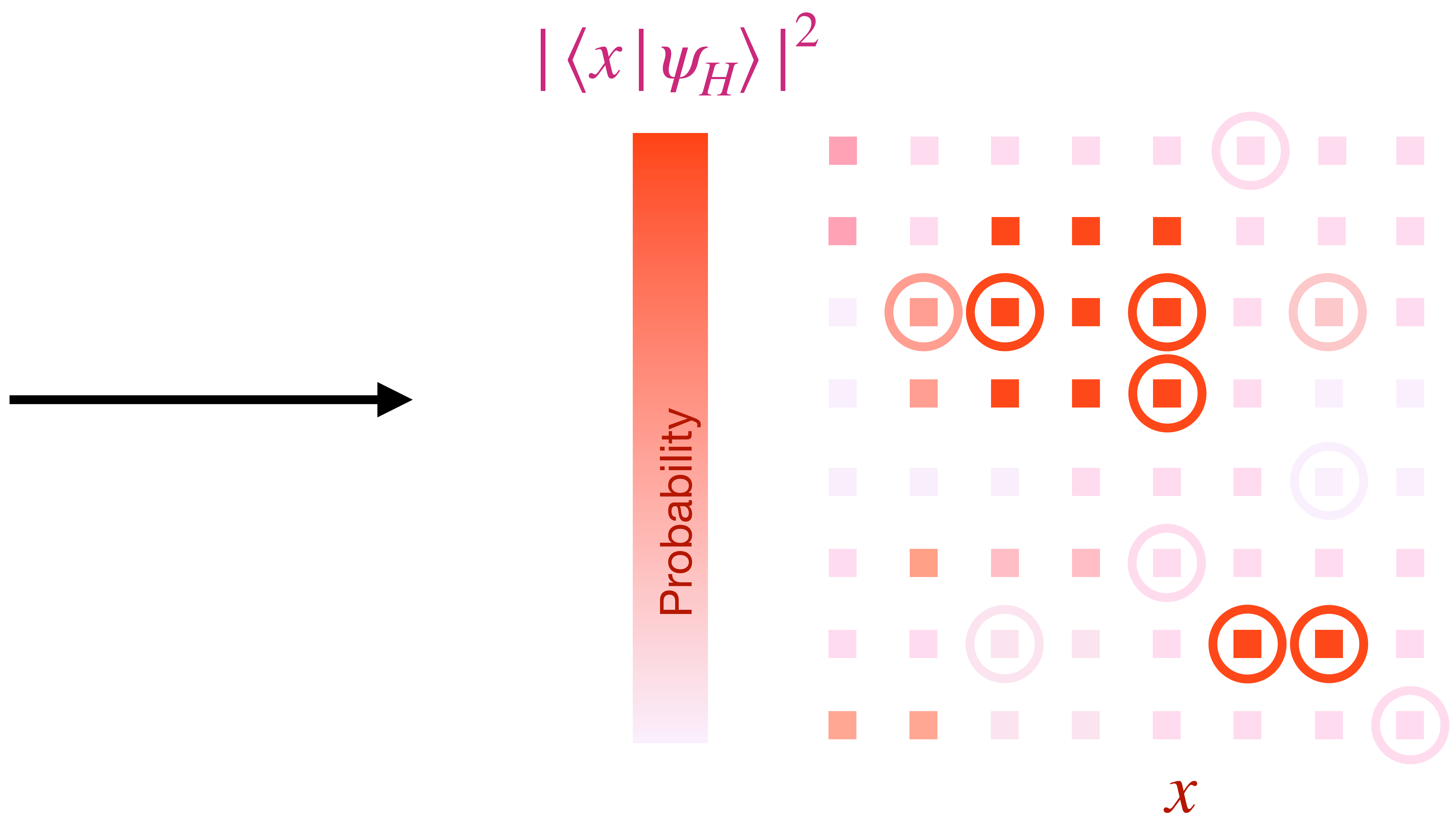
Classical Complexity of GGSP

Can GGSP be **dequantized** as well?

What does $|\psi_{\rm in}\rangle$ mean *classically*?



What does *preparing* $|\psi_H\rangle$ mean *classically*?



Classical Complexity of Stoquastic GGSP

First step: Is GGSP *classically* easy when H is *stoquastic*?

Stoquastic Hamiltonian

$$H = \begin{pmatrix} h_{11} & \leq 0 & \cdots & \leq 0 \\ \leq 0 & h_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \leq 0 \\ \leq 0 & \cdots & \leq 0 & h_{2^m 2^m} \end{pmatrix}$$

In some basis

↓

Perron-Frobenius theorem

$\langle x | \psi_H \rangle \geq 0$ for all x

Hamiltonians without sign problem

Classical optimisation functions - diagonal,

Transverse Field Ising Model,

Quantum architectures: D-Wave, etc.,.

GGSP is *classically* easy for *stoquastic* + *frustration free*

[Bravyi and Terhal'10]

Classical Complexity of Stoquastic GGSP

Is GGSP *classically* easy when H is *stoquastic*?

Gilyén, Hastings and Vazirani; 2021

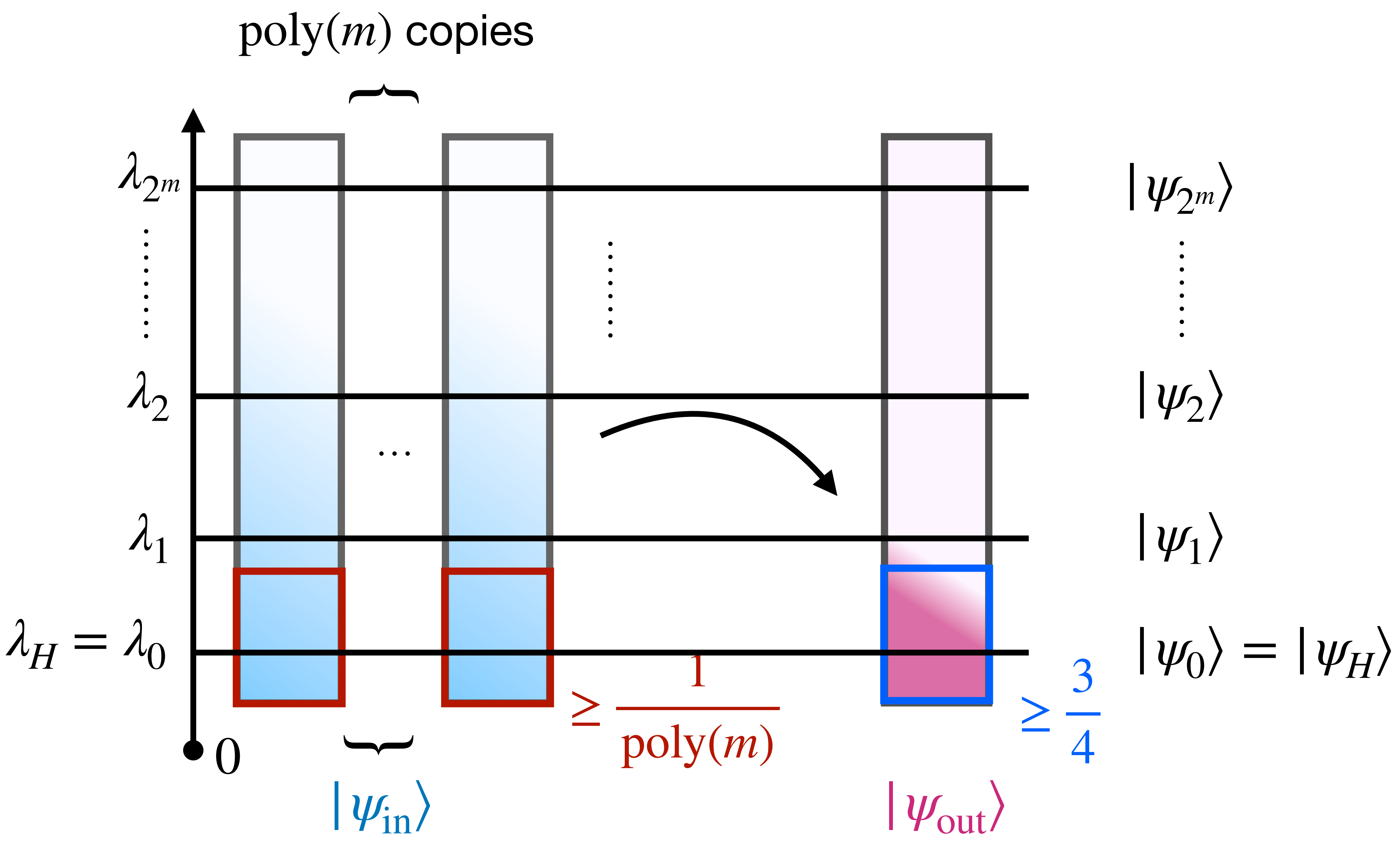
$\exists$ a stoquastic H: *classically* **hard** to prepare $|\psi_H\rangle$ given a **particular** $|\psi_{\text{in}}\rangle$ whereas it is *quantumly* easy to prepare $|\psi_H\rangle$

Our result

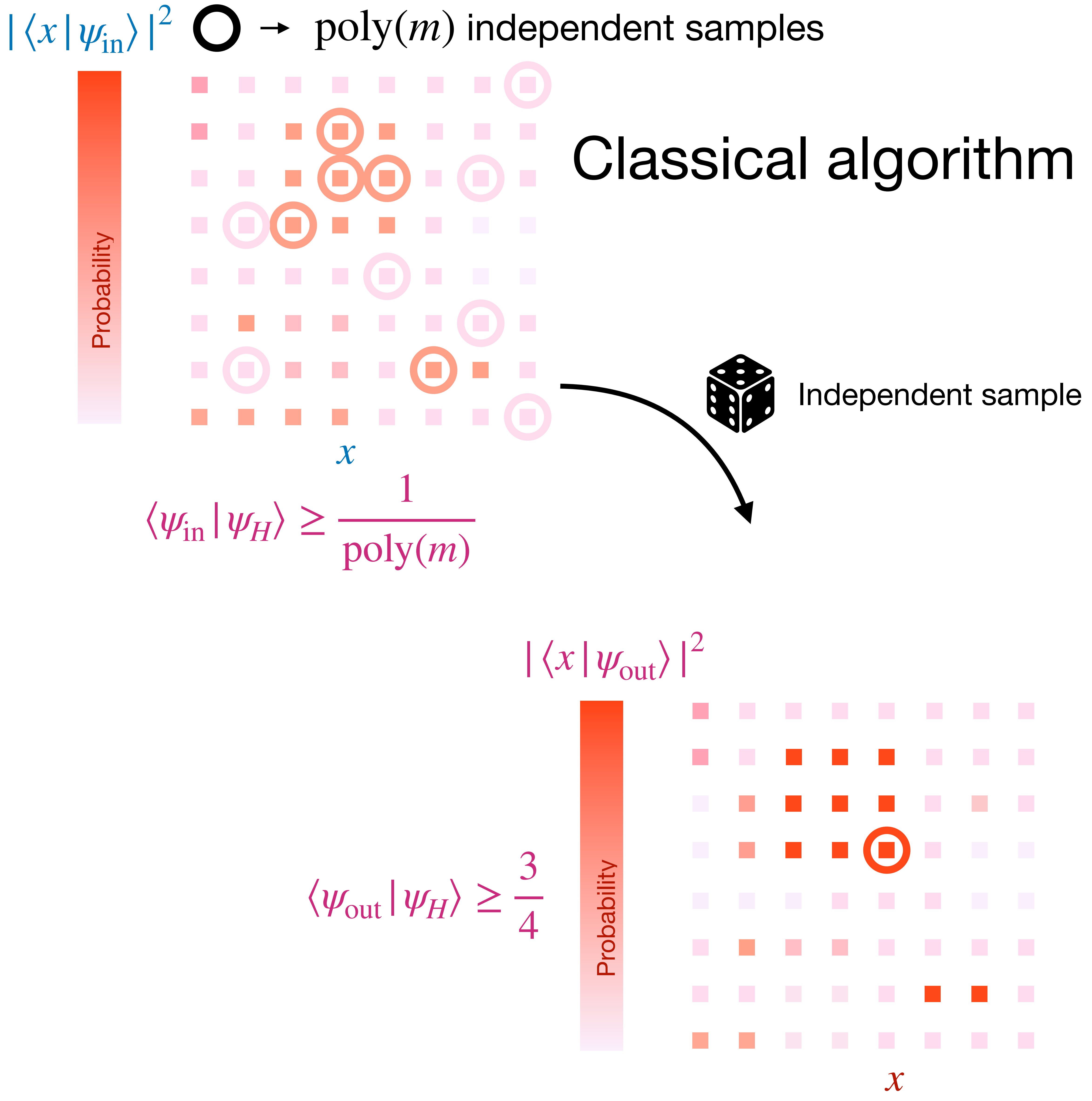
$\exists$ a stoquastic H: *classically* **hard** to prepare $|\psi_H\rangle$ given an **arbitrary** $|\psi_{\text{in}}\rangle$ whereas it is *quantumly* easy to prepare $|\psi_H\rangle$

*Both hardness results are in terms of number of queries to a resource

Formal problem statement



Quantum algorithm



Classical algorithm

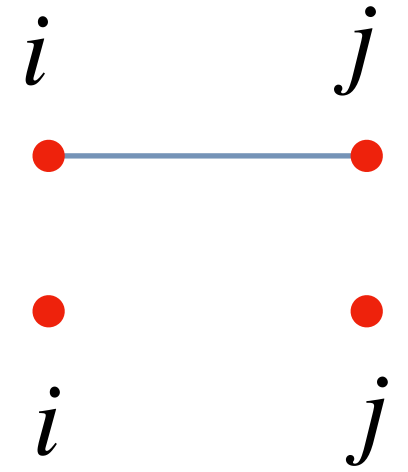
Proof sketch: Construction of H

Expander E - large *girth* and large *gap*

Locally tree like!

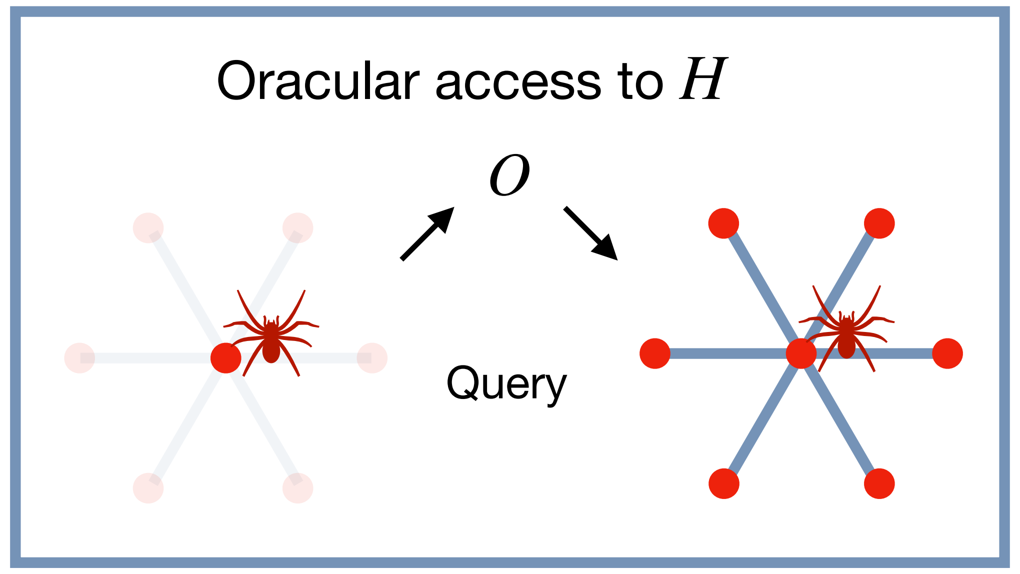
$$H = -A_G$$

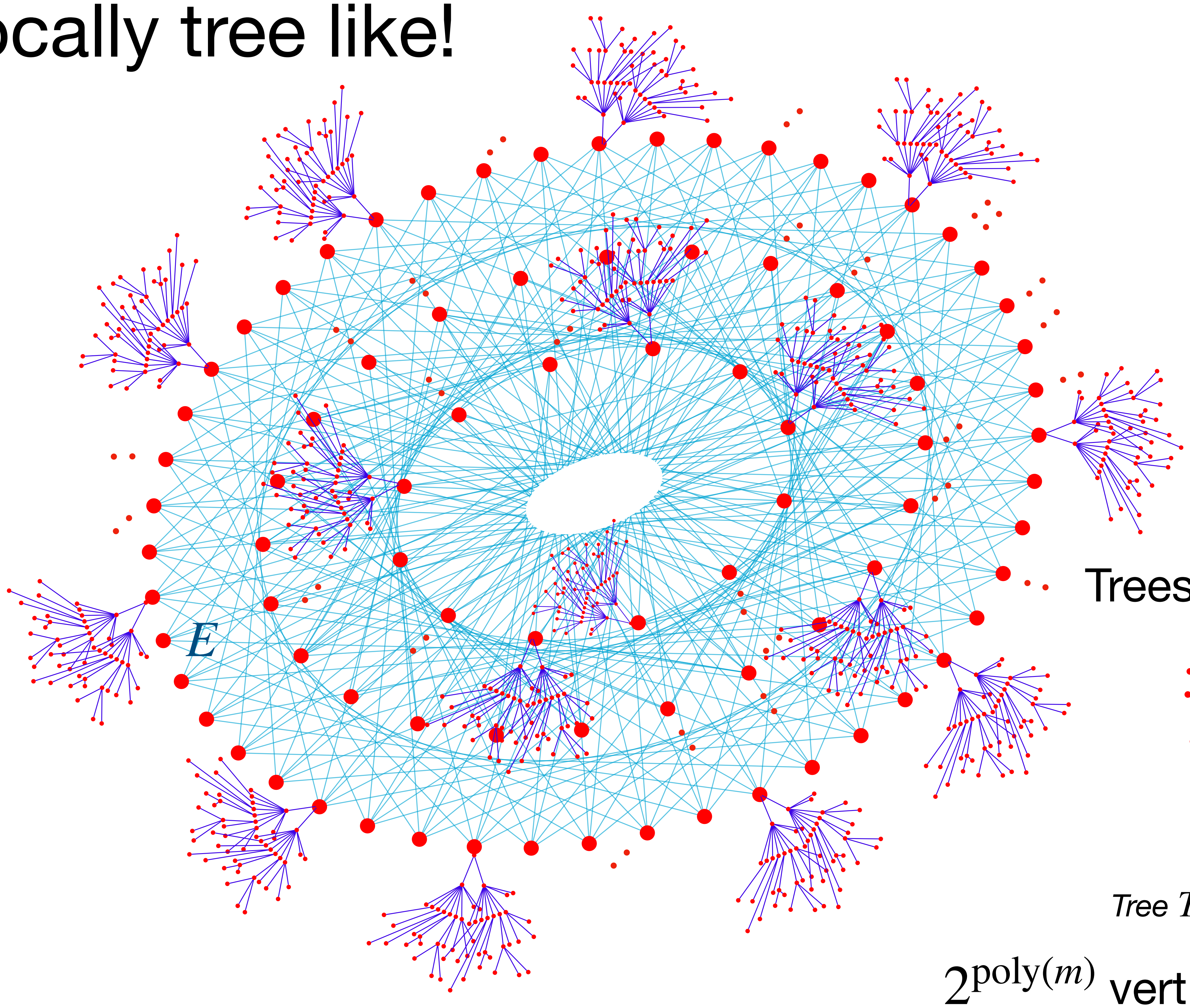
$$\langle i | H | j \rangle = \begin{cases} -1 & \text{if} \\ 0 & \text{if} \end{cases}$$



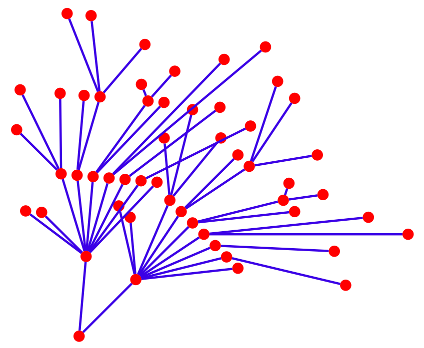
Stoquastic!

$|\psi_H\rangle$ = top eigenvector of A_G





Trees match local picture of E



Tree T - On all vertices

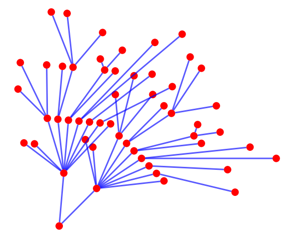
Hard/Easy - More/Less queries to O

$2^{\mathrm{poly}(m)}$ vertices in T as well

$2^{\mathrm{poly}(m)}$ vertices $\quad G$

∃ a *quantum* algorithm for GGSP performing $\mathrm{poly}(m)$ queries

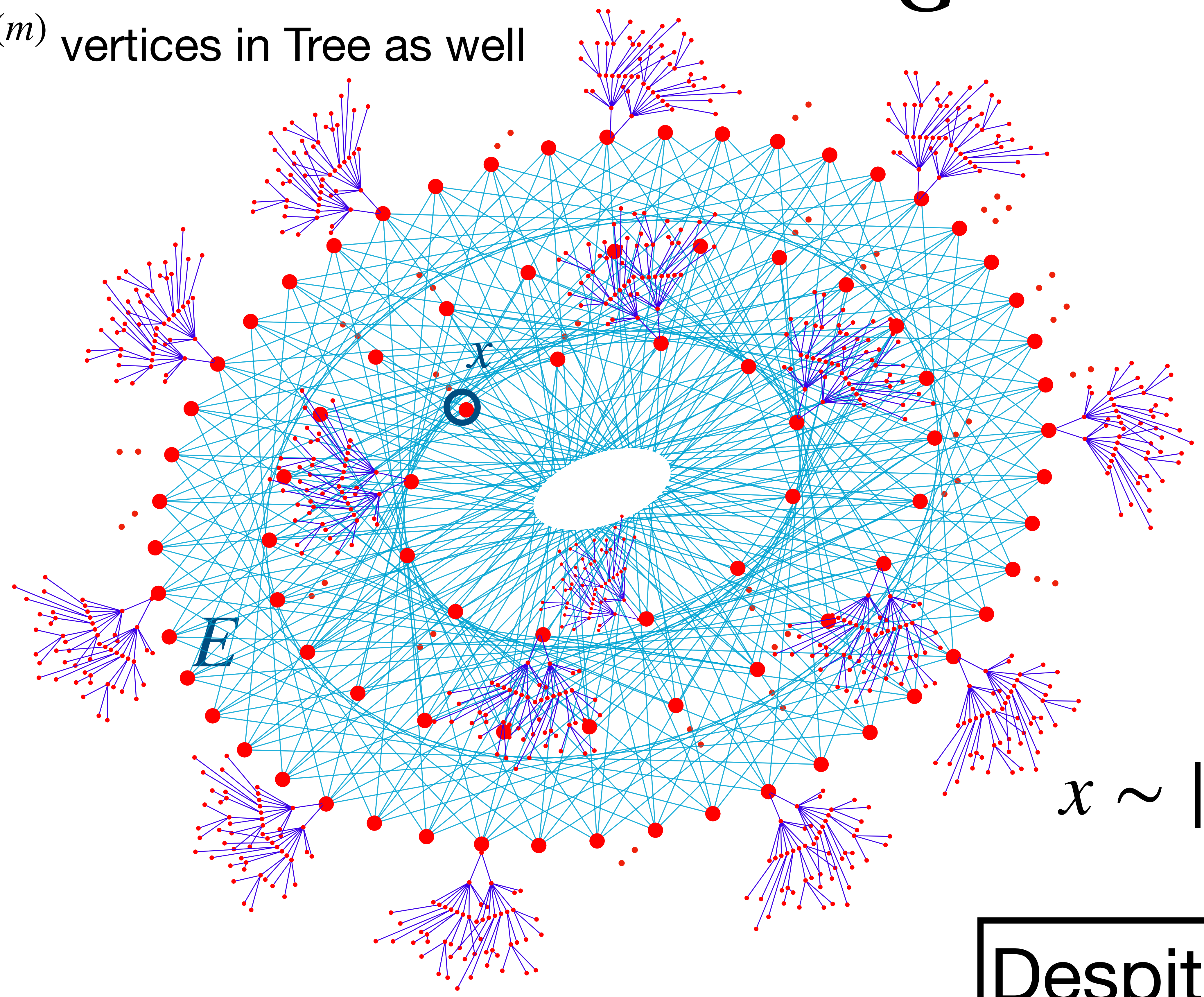
Concentration property of $|\psi_H\rangle$



Tree on all vertices

$2^{\mathrm{poly}(m)}$ vertices in Tree as well

$$H = -A_G$$



Concentration Property of $|\psi_H\rangle$

$|\langle \mathrm{Trees}|\psi_H\rangle|^2 = o(1)$ Very tiny

$|\langle \mathrm{Expander}|\psi_H\rangle|^2 = 1 - o(1)$ Very large

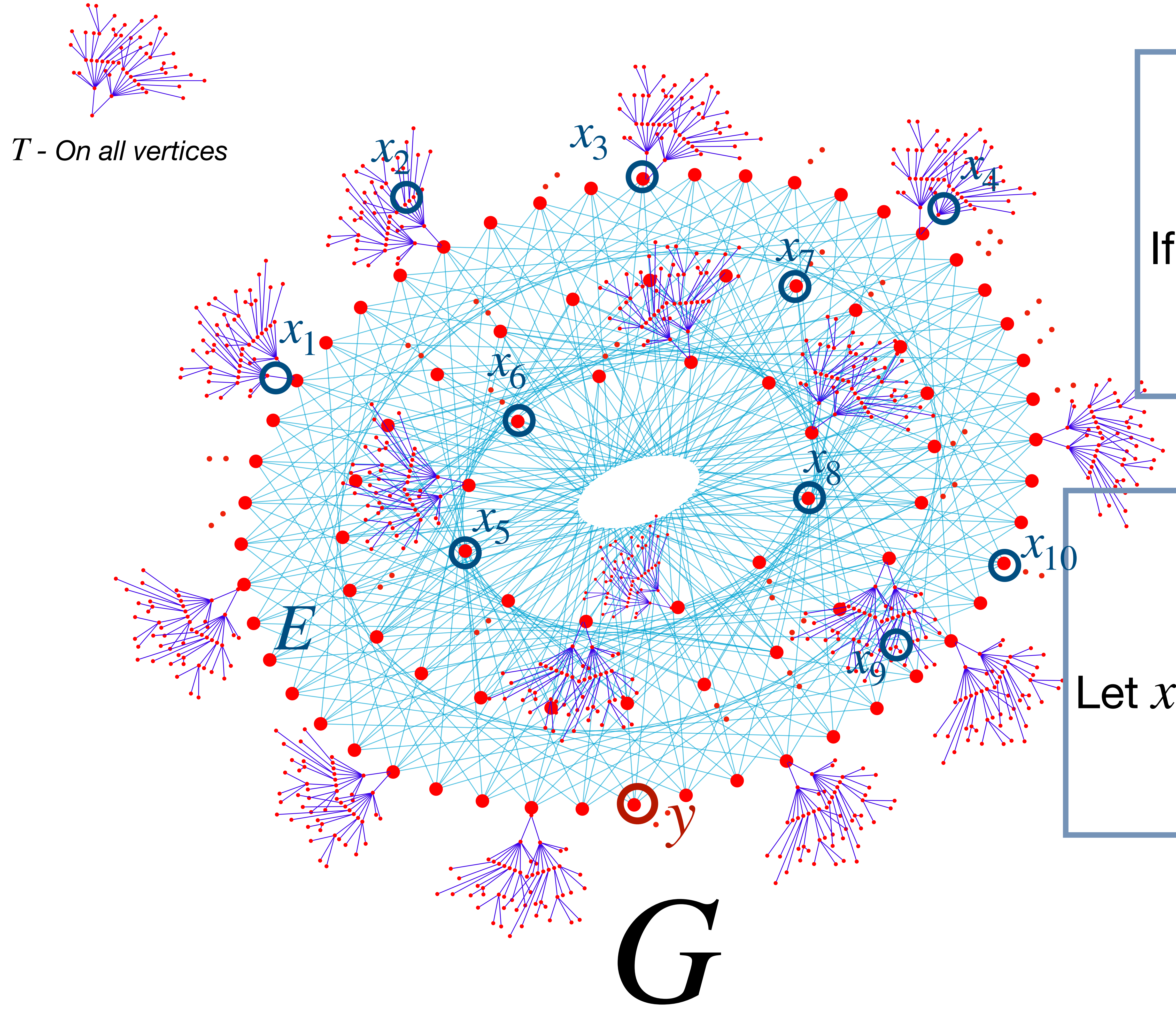
$\downarrow$

$x \sim |\langle x|\psi_H\rangle|^2 \implies x \in$ Expander, with high probability

Despite each **tree** having **huge** number of vertices!

$2^{\mathrm{poly}(m)}$ vertices G

Proof sketch: Localization



Note that this is for all guiding states!

Localization Property of $|\psi_H\rangle$

If $x_1, x_2 \ldots x_t \sim \left(|\langle x_i | \psi_H \rangle|^2 \right)^{\otimes t}$, then they are ***far*** from each other with high probability

Localization Property for **all** $|\psi_{\text{in}}\rangle, |\psi_{\text{out}}\rangle$

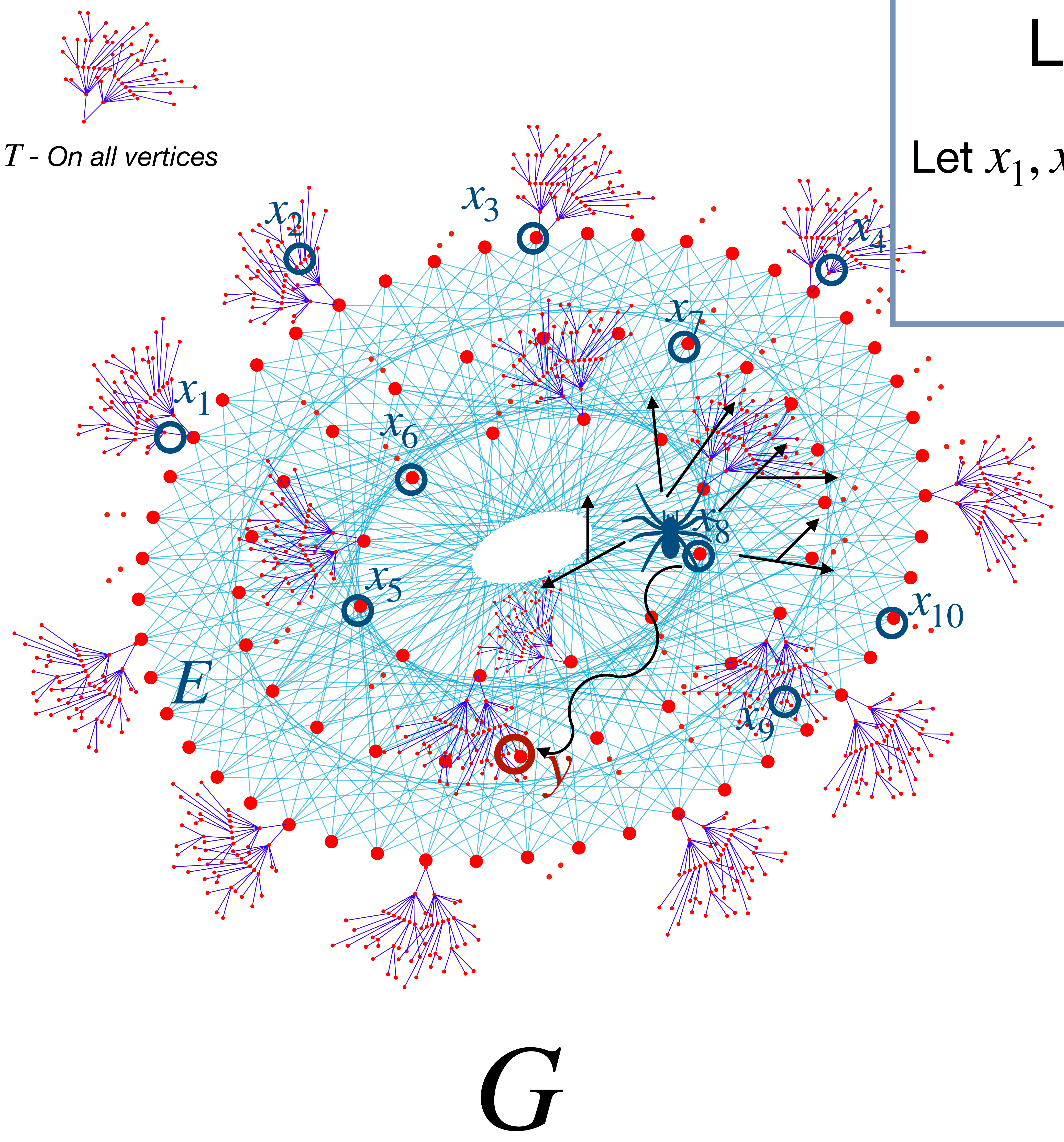
Let $x_1, x_2 \ldots x_t \sim \left(|\langle x_i | \psi_{\text{in}} \rangle|^2 \right)^{\otimes t}$. If $y \sim |\psi_{\text{out}}\rangle$, then y is ***far*** from all x_is with high probability

↓

Any classical algorithm solving GGSP must ***output*** a y ***far*** from all x_is

with high probability

Proof sketch: Localization



Localization Property for **any** $|\psi_{\text{in}}\rangle, |\psi_{\text{out}}\rangle$

Let $x_1, x_2 \ldots x_t \sim \left(|\langle x_i | \psi_{\text{in}} \rangle|^2 \right)^{\otimes t}$. If $y \sim |\psi_{\text{out}}\rangle$, then y is ***far*** from all x_is

with high probability

But, only oracle access to H

Classical algorithms can perform only ***local exploration of*** G from x_i s

⬇

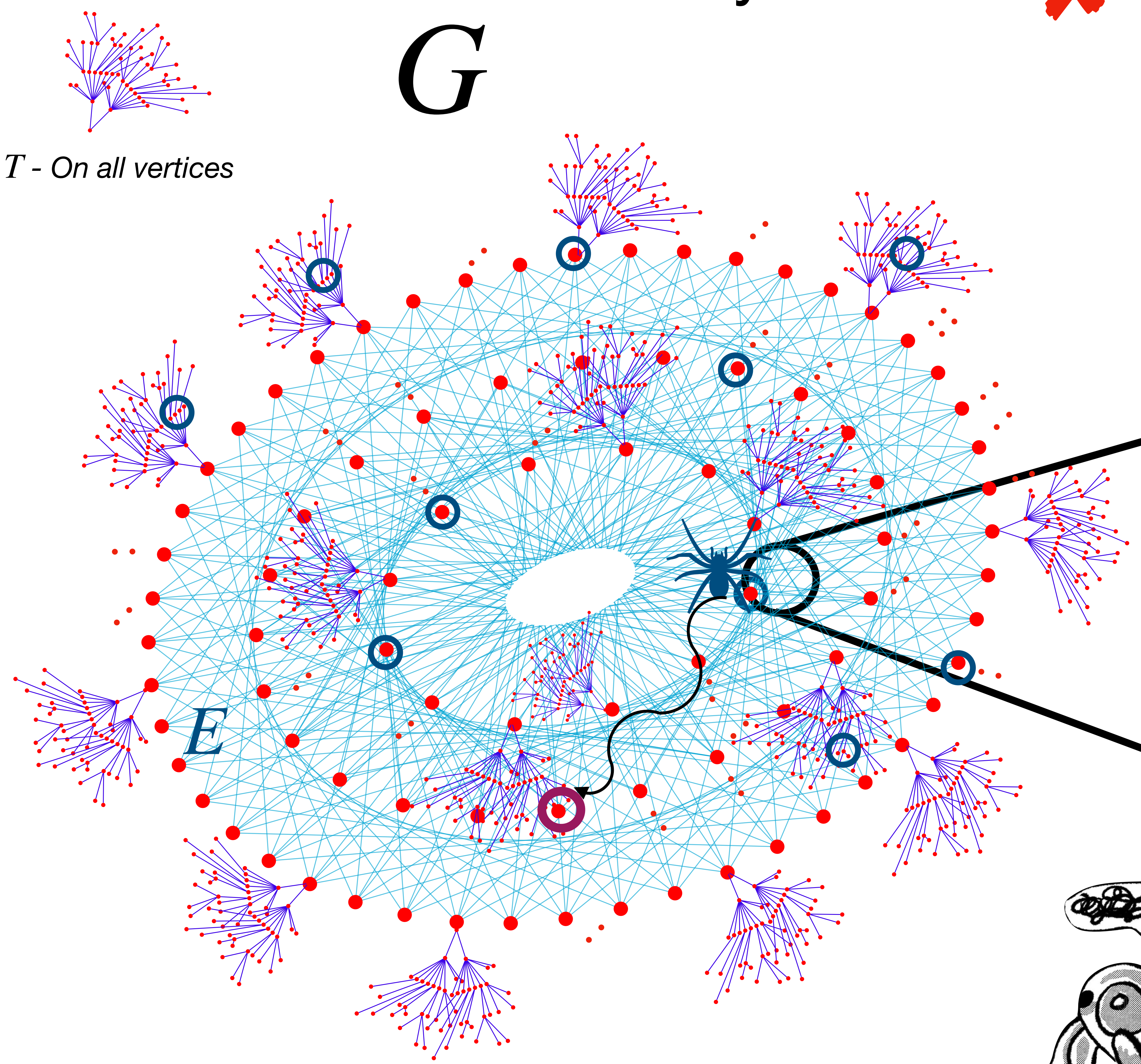
Any classical algorithm solving GGSP must ***explore*** a y ***far*** from all x_is

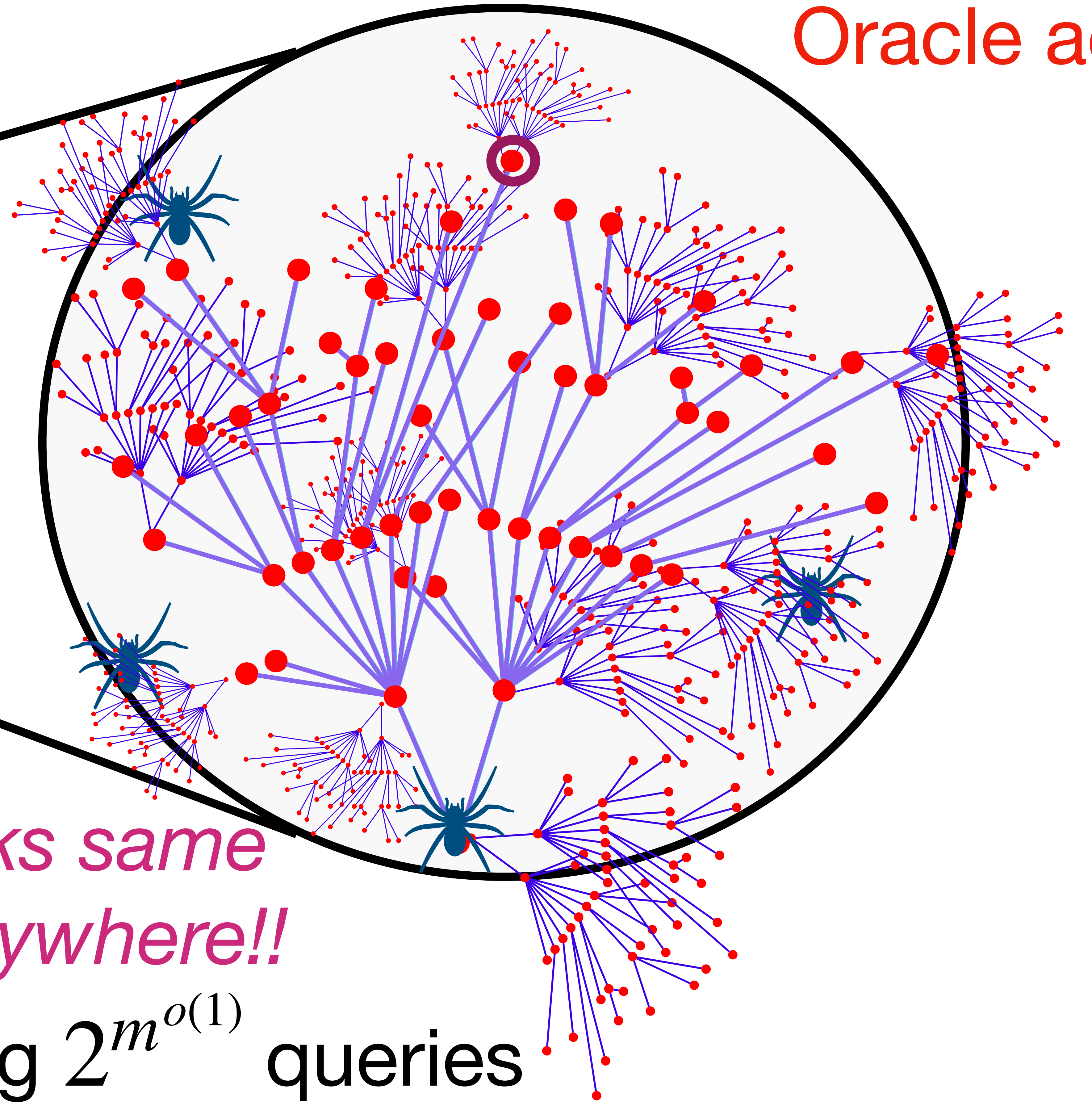
with high probability

Proof sketch: How difficult is it to find a far y?

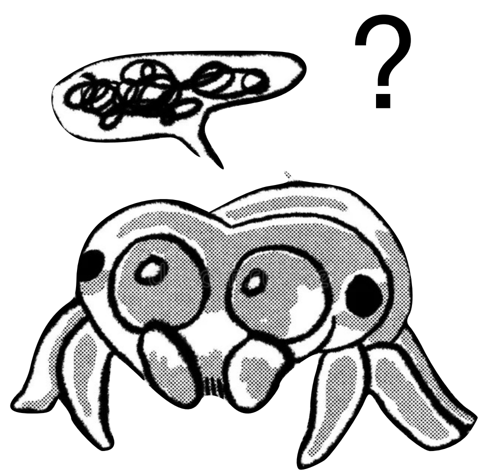
This is very difficult! ✘

Any classical algorithm solving GGSP must ***explore*** a y ***far*** from all x_is





Oracle access!



Looks same everywhere!!

Very small probability of exploring y even after doing $2^{m^{o(1)}}$ queries

Conclusion

Our result

$\exists$ a stoquastic H: *classically* **hard** $\left(2^{m^{o(1)}}\right)$ to prepare $|\psi_H\rangle$ given an **arbitrary** $|\psi_{\text{in}}\rangle$ whereas it is *quantumly* easy $(\text{poly}(m))$ to prepare $|\psi_H\rangle$

Rules out *dequantized* algorithms for GGSP given ***arbitrary*** $|\psi_{\text{in}}\rangle$

Rules out *dequantization* of *adiabatic paths* that go through our H

Open question 1

Is there a stoquastic H: classically hard $(2^{O(m)})$ to prepare $|\psi_H\rangle$ but easy $(\text{poly}(m))$ to prepare $|\psi_H\rangle$ quantumly ?

Open question 2

What is the classical complexity of GGSP for local H?